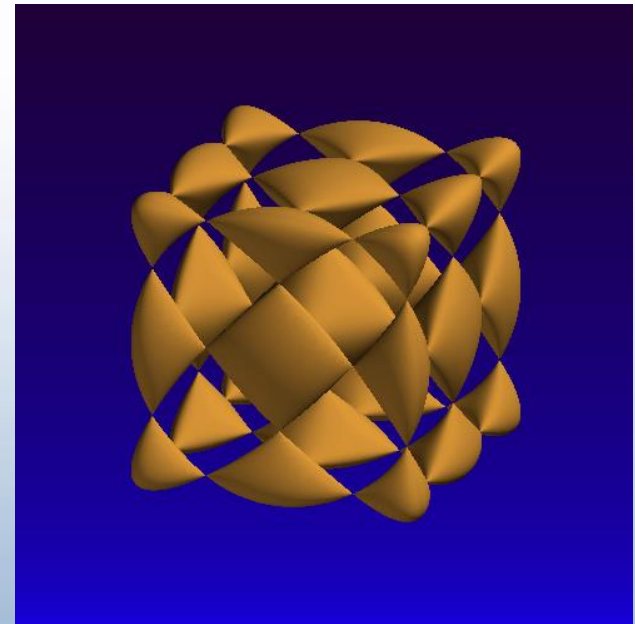




# *Implicit Objects*



<http://hm.softalliance.net/>

*Alexander Pasko, Evgenii Maltsev*



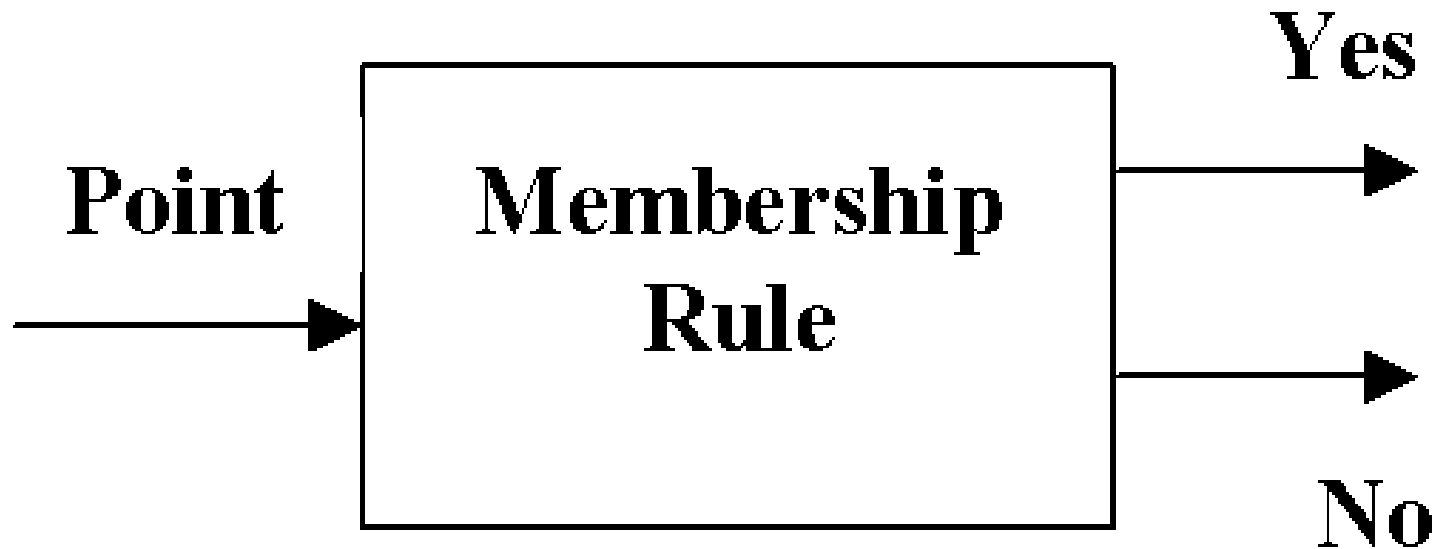
# Contents

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- *Point membership rule*
- *Implicit curves*
- *Algebraic surfaces*
- *Skeletal implicit surfaces*
- *Convolution surfaces*



# Point Membership Rule



Implicit curves and surfaces



# Implicit Curves and Areas

---

A set of points on a 2D plane with

$$f(x, y) = 0$$

is called an implicit curve

A 2D area (piece of plane, planar area, 2D solid) is defined as

$$f(x, y) \geq 0$$

with the implicit curve as its boundary.

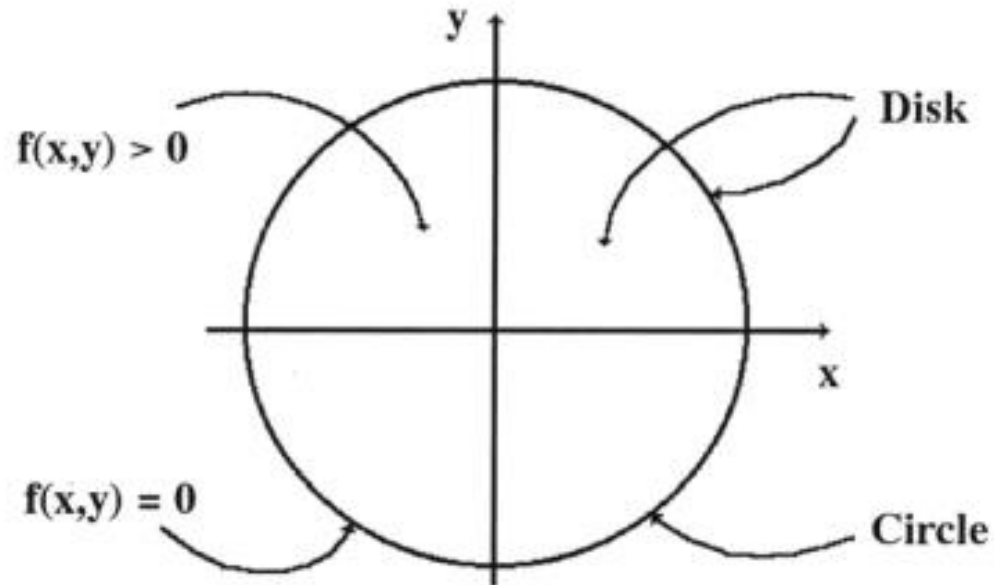


# Circle and Disk

$$f(x,y) = R^2 - x^2 - y^2$$

**Disk (k=2)**  $f(x,y) \geq 0$

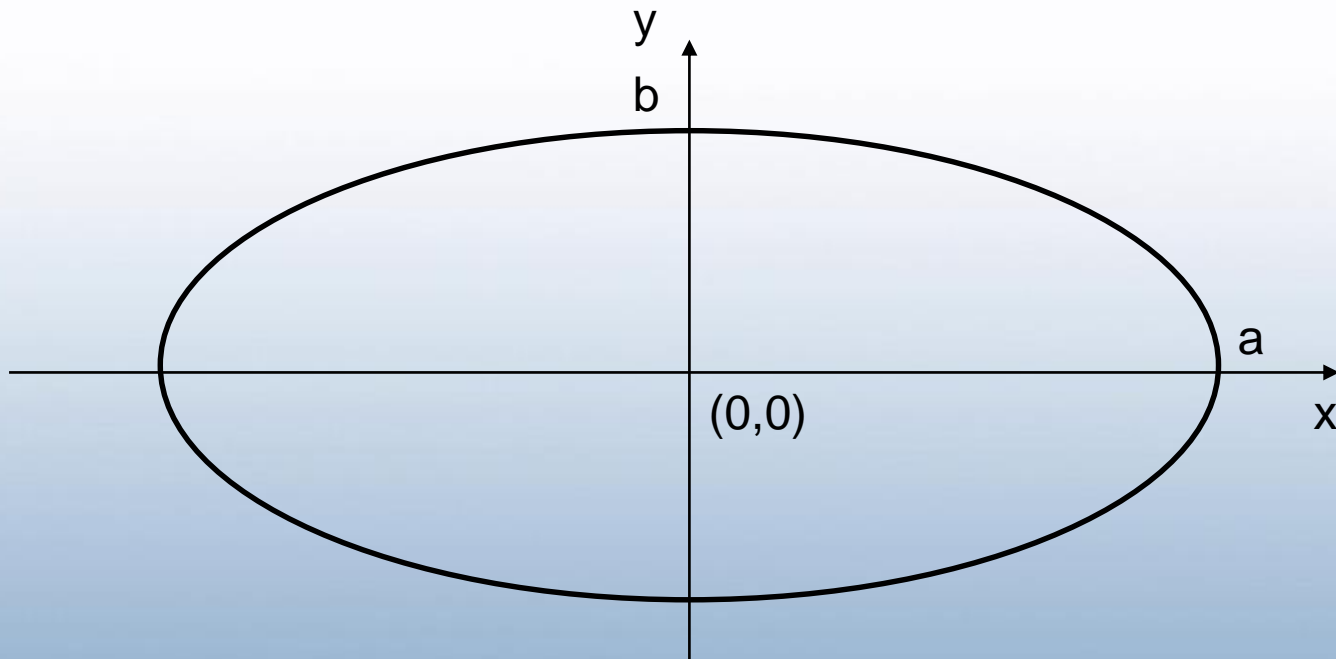
**Circle (k=1)**  $f(x,y) = 0$





# Ellipse

$$F(x,y) = 1 - (x/a)^2 - (y/b)^2$$

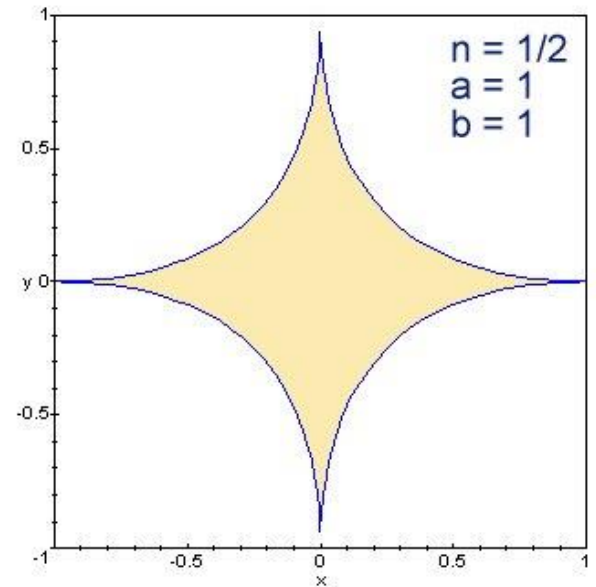
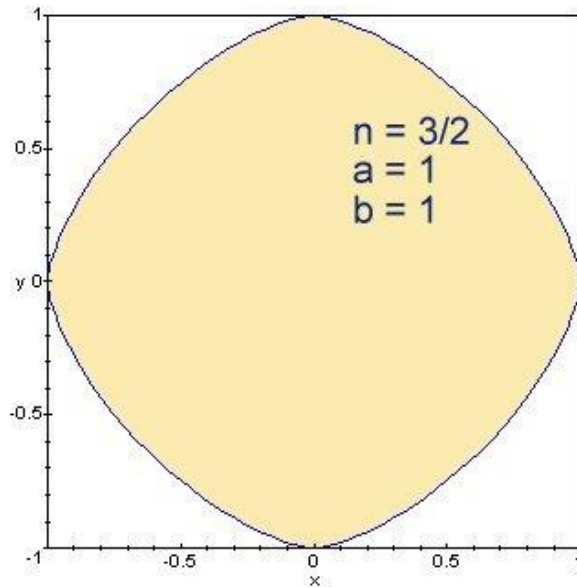
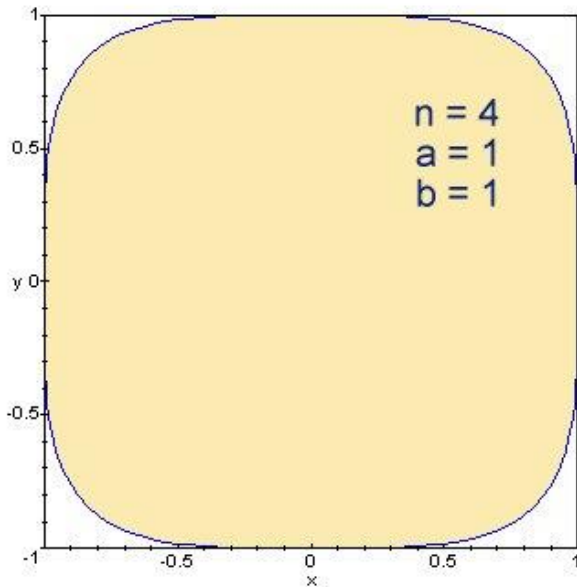


$$F(x,y) = 1 - ((x-x_0)/a)^2 - ((y-y_0)/b)^2 \quad ?$$



# Superellipse

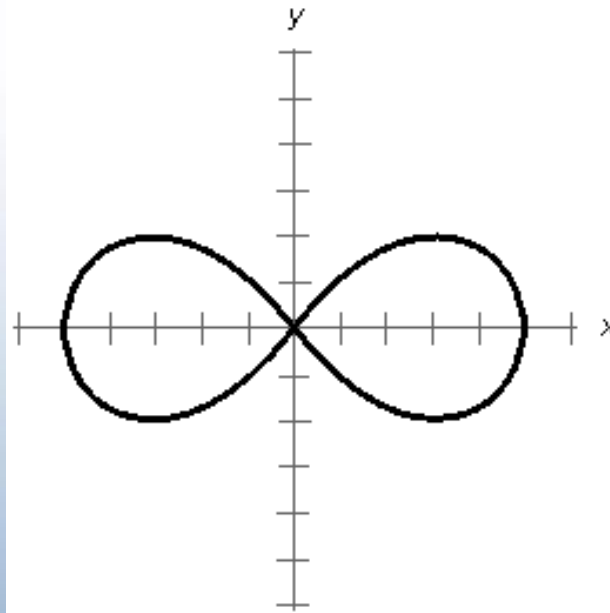
$$F(x,y) = 1 - (x/a)^n - (y/b)^n$$





# Lemniscate of Bernoulli

$$(x^2 + y^2)^2 = a^2(x^2 - y^2)$$



The lemniscate was first described in 1694 by Jakob Bernoulli as a modification of an ellipse. The curve has become a symbol of infinity and is widely used in math.







# “Implicit” Surfaces

---

A set of points with

$$f(x, y, z) = 0$$

is called an “implicit” surface

Is the term “implicit” correct?



# Explicit or “Implicit”?

---

Two ways to define  $z$  as a function of  $x, y$ :

1) Explicit form

$$z = \phi(x, y)$$

2) Implicit form

$$f(x, y, z) = 0$$

Both forms are **not enough**  
to define a sphere,  
**only** a hemi-sphere.



# Iso-valued surface

---

$$\xi = f(x, y, z)$$

is an explicit function of three variables

$$\xi = 0 \text{ or } f(x, y, z) = 0$$

**is an iso-valued surface (*isosurface*) of a function of three variables**

(or an “implicit” surface as the historical accident)



“Implicit” Surface - OK

Implicit Function - NO



# 3D Solids

---

A 3D solid is defined as

$$f(x, y, z) \geq 0$$

Isosurface  $f=0$  or “implicit” surface is the boundary of this solid.



# Classes of implicit surfaces

---

- Algebraic surfaces
- Skeletal implicit surfaces
- Convolution surfaces



# Algebraic Surfaces

---

Surfaces with polynomial  $f(x,y,z)$

- Quadratic
  - Sphere
  - Ellipsoid
  - Cylinder
  - Cone
  - Paraboloid, ...
- Torus
- Superellipsoids



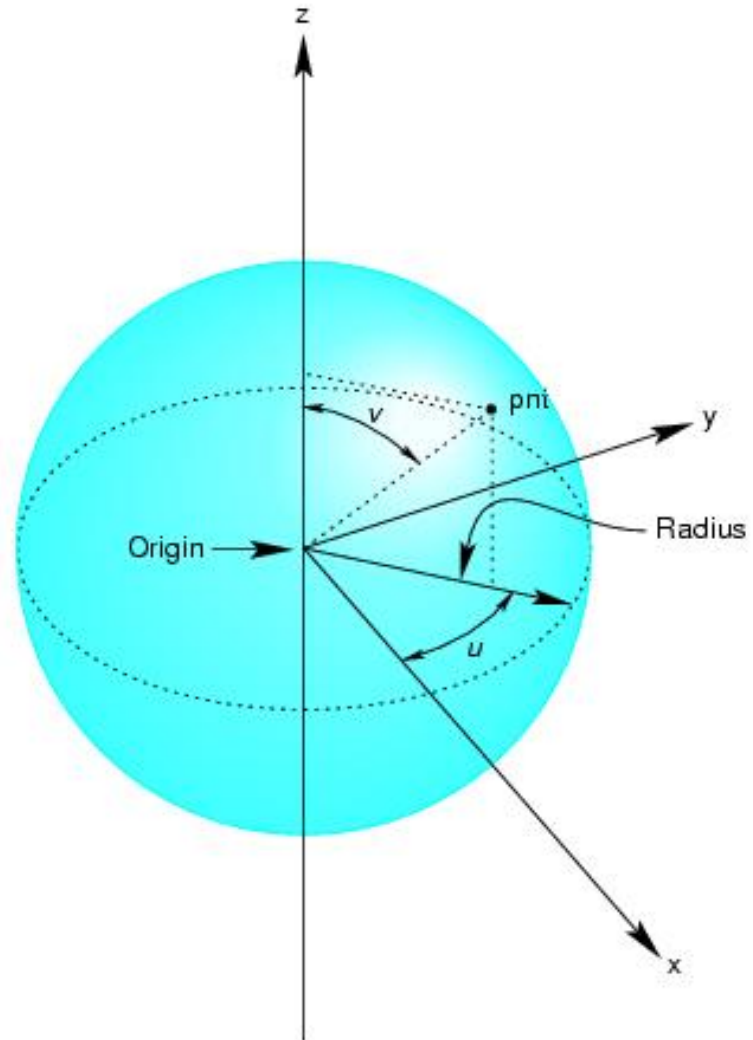
# Sphere and Solid Ball

Sphere surface:

$$R^2 - x^2 - y^2 - z^2 = 0$$

Solid ball:

$$R^2 - x^2 - y^2 - z^2 \geq 0$$

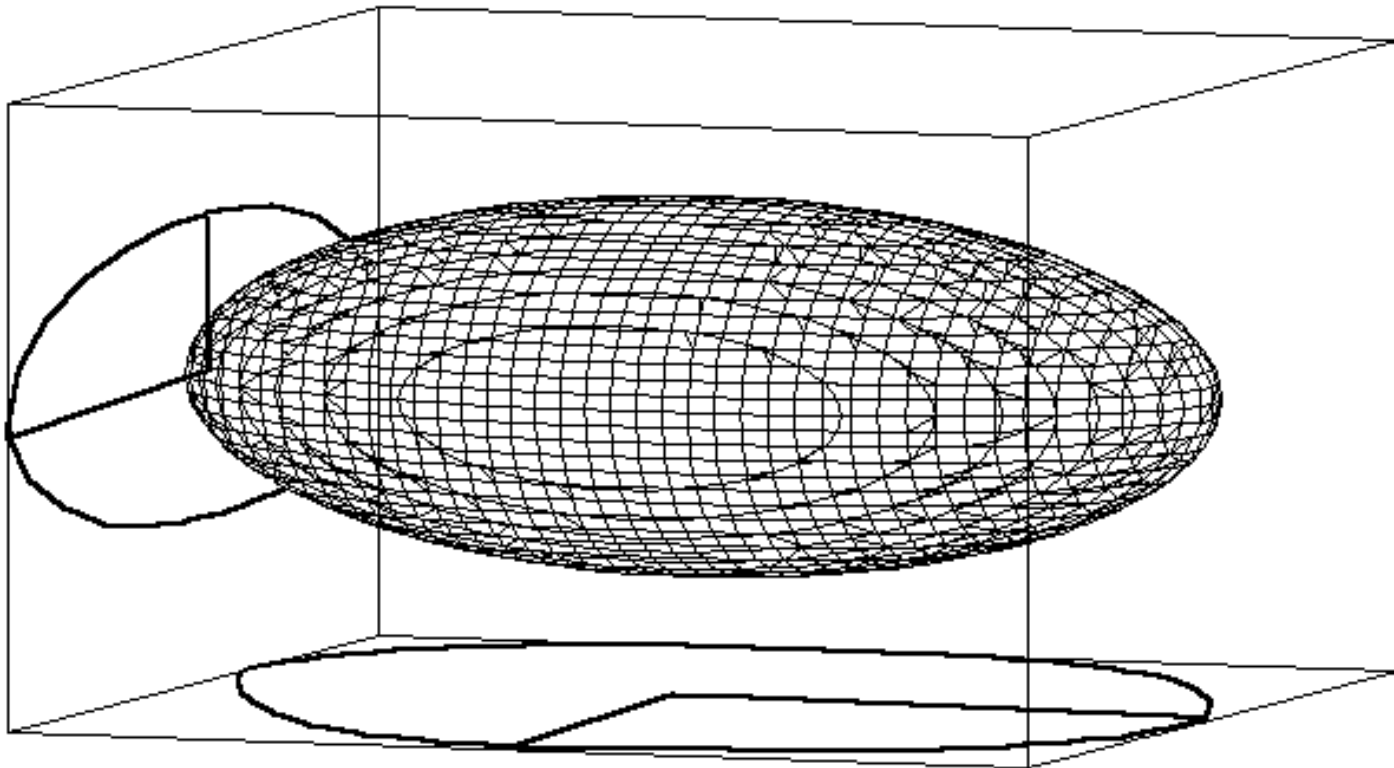






# Ellipsoid

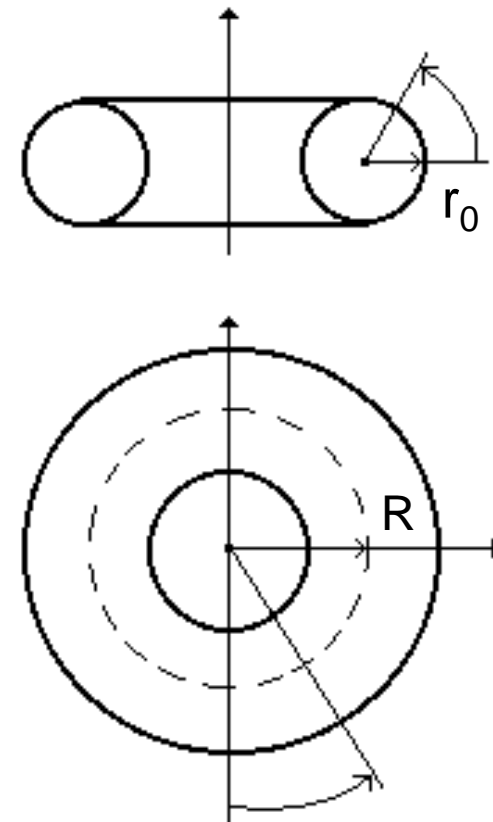
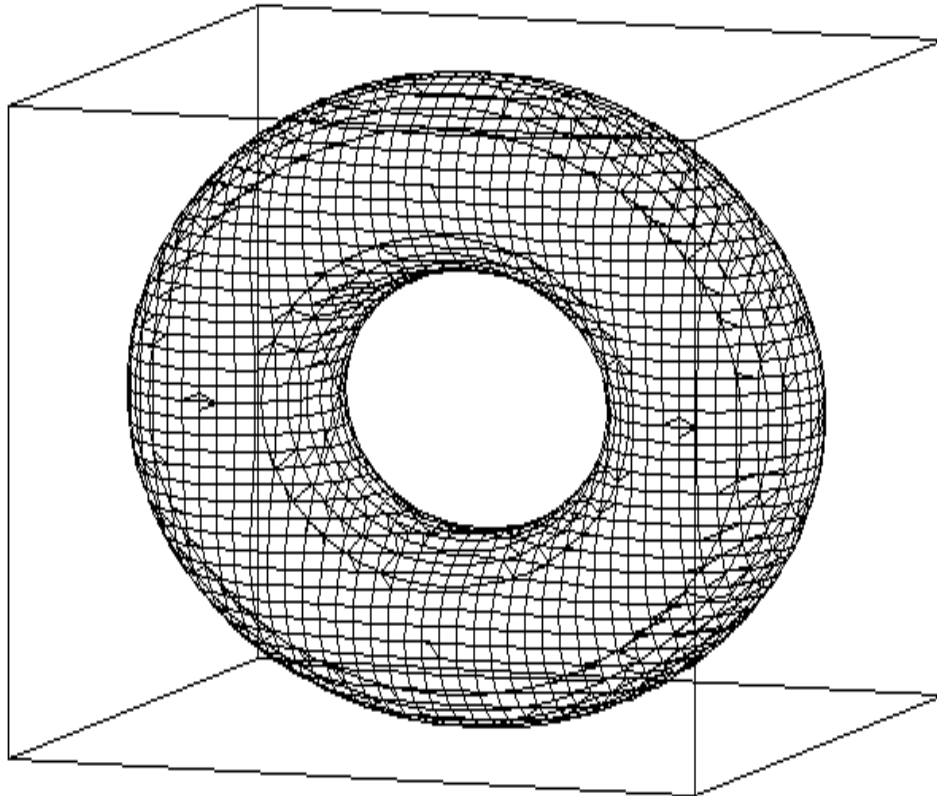
$$F(x,y,z) = 1 - (x/a)^2 - (y/b)^2 - (z/c)^2$$





# Torus

$$F(x,y,z) = r_0^2 - x^2 - y^2 - z^2 - R^2 + 2R \sqrt{x^2 + y^2}$$





# Superellipsoids

$$f(x, y, z) = 1 - \left[ \left( \frac{x}{r_x} \right)^{\frac{2}{s_2}} + \left( \frac{y}{r_y} \right)^{\frac{2}{s_2}} \right]^{\frac{s_2}{s_1}} - \left( \frac{z}{r_z} \right)^{\frac{2}{s_1}}$$

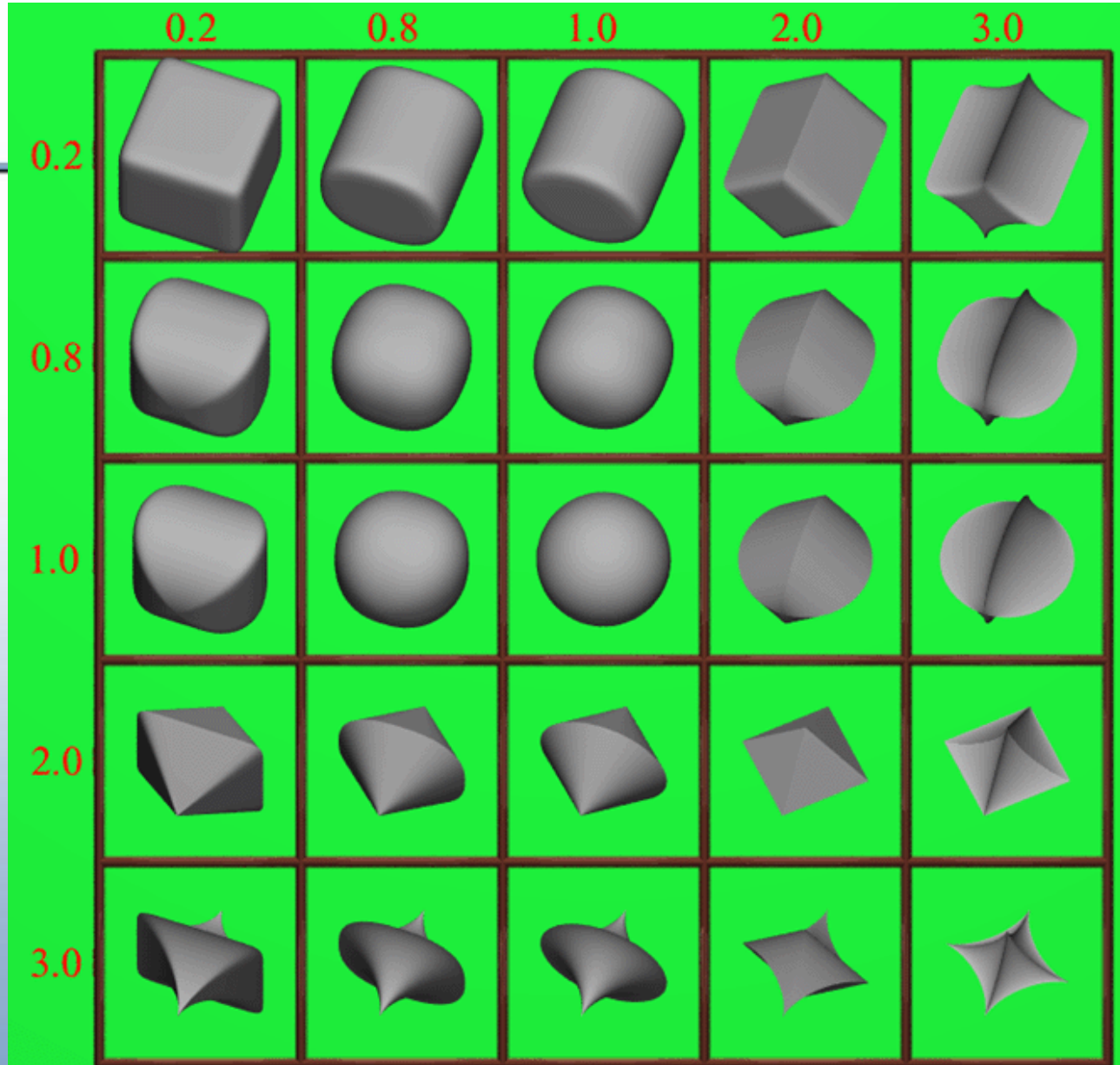
$s_1$  and  $s_2$  – shape control parameters



$S_2$

Superellipsoids

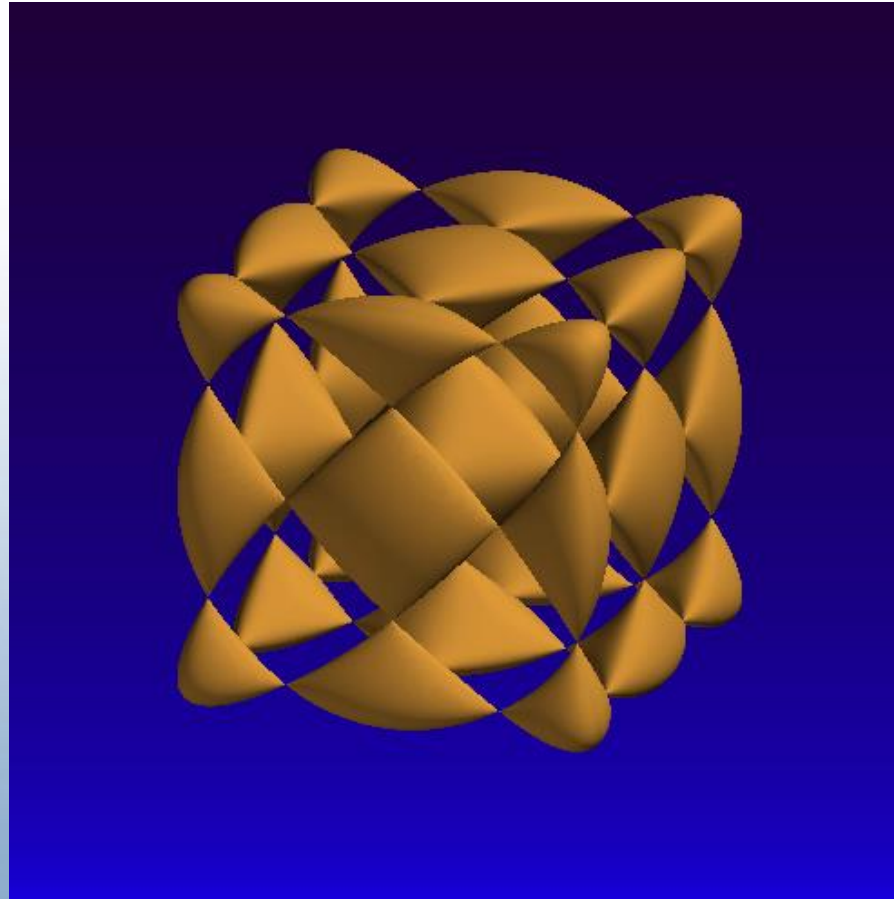
$S_1$





# Chebyshev Polynomial

---

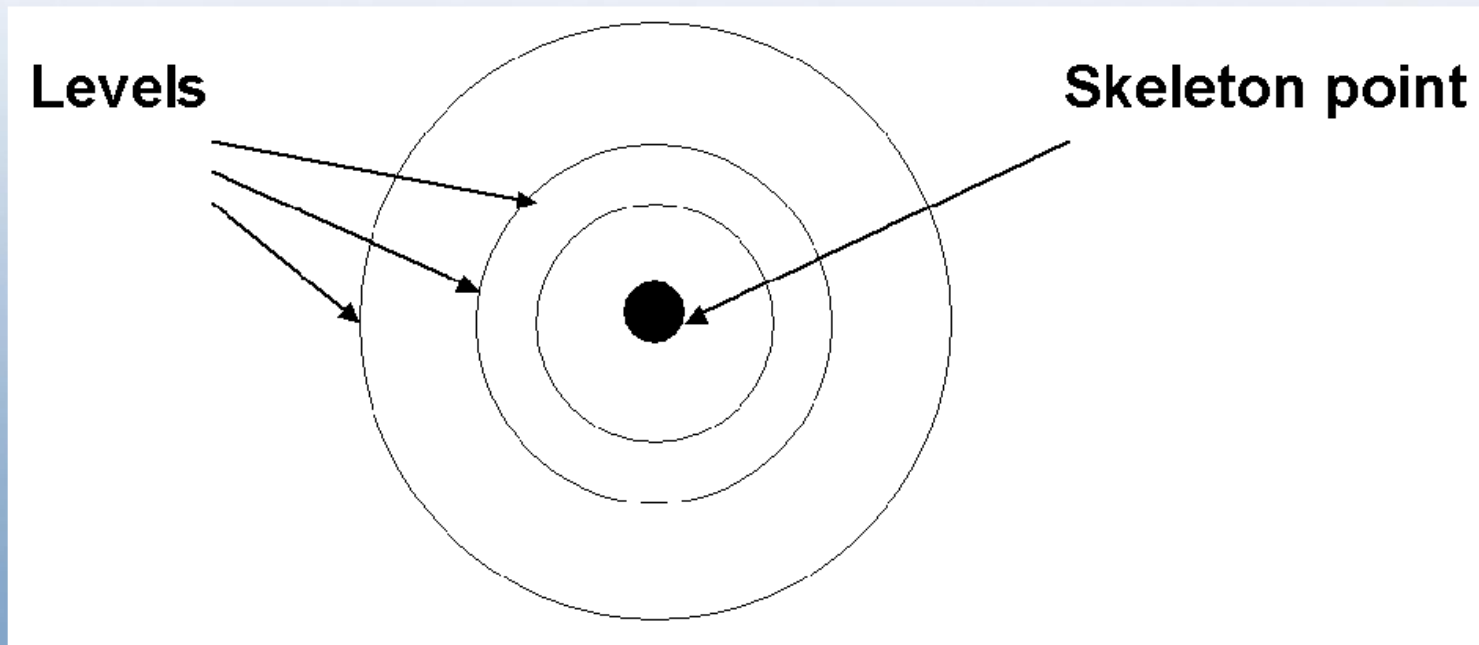




# Skeletal Implicit Surfaces

Blinn [1982]:

modeling isosurfaces as  
a side effect of visualizing  
electron density fields





## **Skeletal model elements:**

- *Skeleton* (points, lines and others);
- *Scalar field* with an individual skeleton element as a source;
- *Global field* as an *algebraic sum* of individual fields;
- *Level* (or *threshold*) of the field value defining the isosurface of interest.

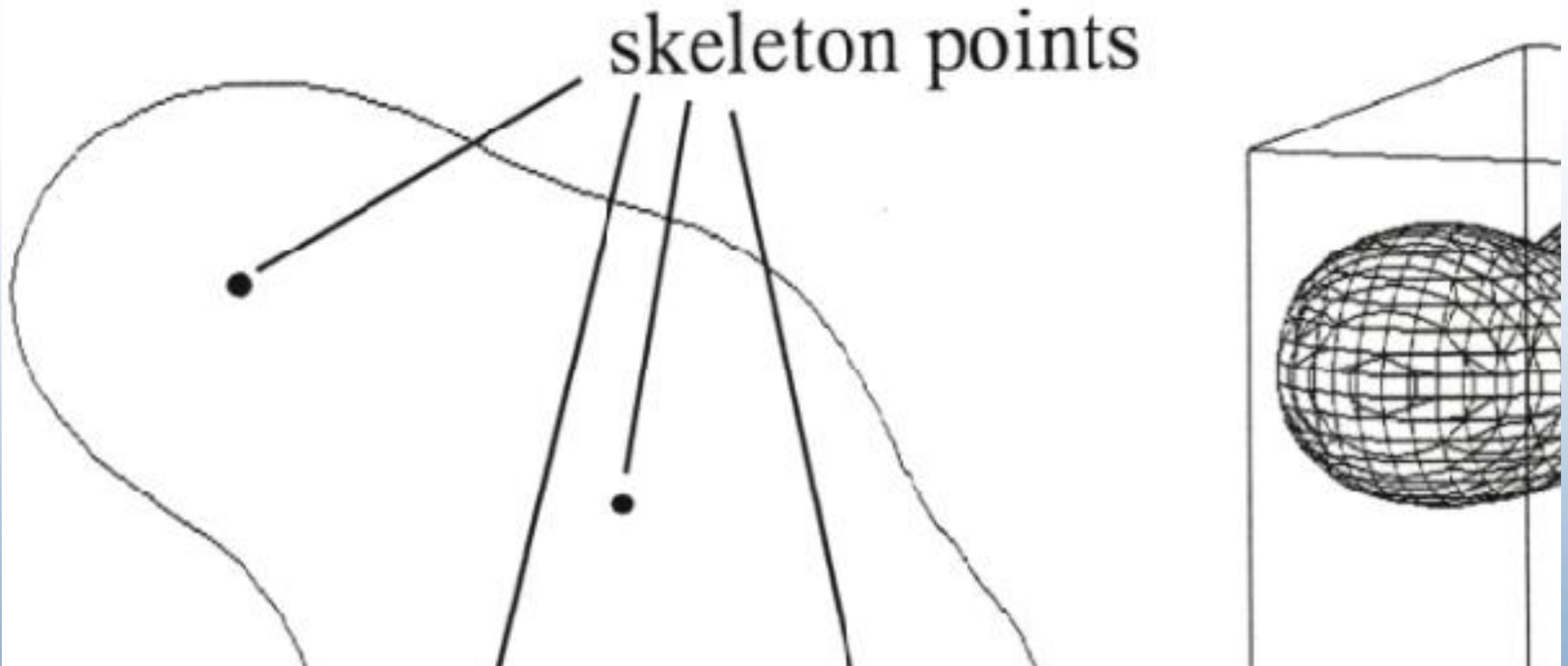
# Skeletal Implicit Surfaces



2D isoline

3D isosurface

skeleton points







# Skeletal Surface Definition

---

$$F(P) - T = 0$$

with 
$$F(P) = \sum_{i=1}^N c_i F_i(r_i)$$

$N$  is the number of skeletal elements,

$F_i$  is the individual scalar field,  
(*blending function*) of the  $i$ -th element,

$r_i$  is the distance from  $P$  to the  $i$ -th element,

$T$  is the *threshold* (or *level value*).

Bloppy models, metaballs, soft objects



# Blobby Model

Blinn [1982]:

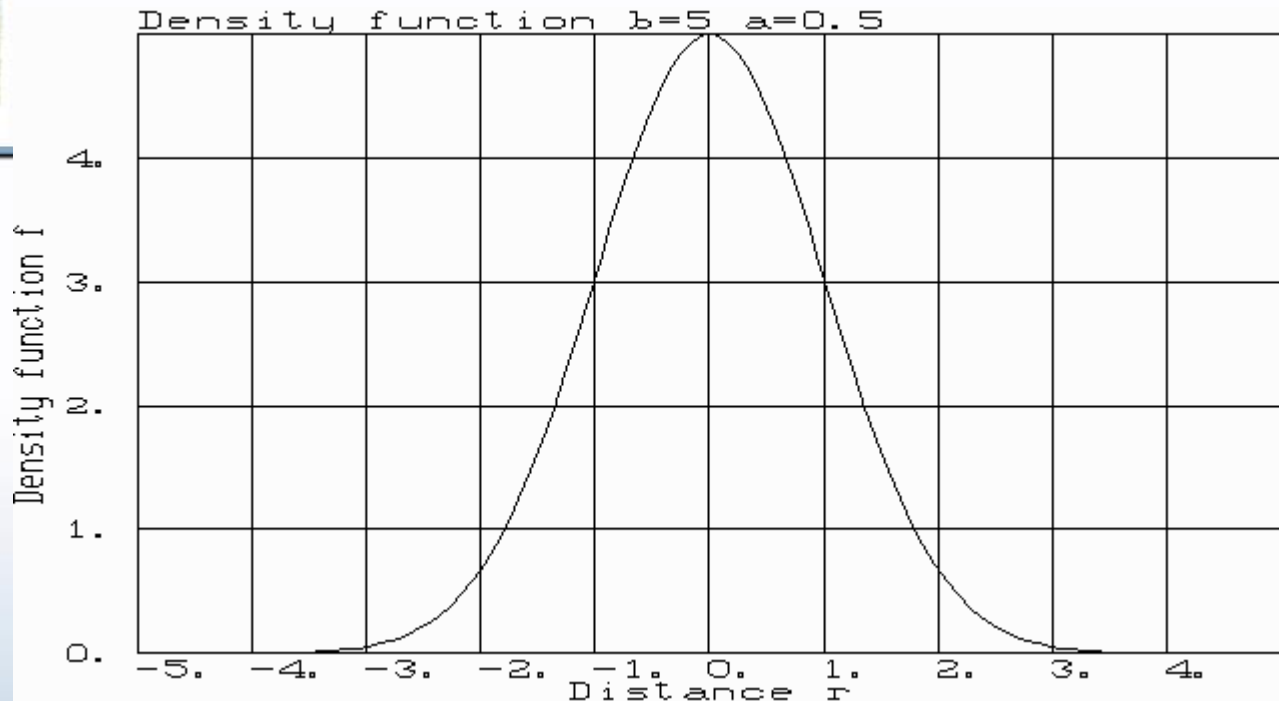
$$F_i(r_i) = b_i e^{-a_i r_i^2}$$

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

$(x, y, z)$  are coordinates of the given point P,

$(x_i, y_i, z_i)$  are coordinates of the  $i$ -th skeleton point

# Bloppy model



## Properties:

- Exponential field does not fall to zero;
- All elements contribute to the field in any point (global influence);



# Metaballs

Nishimura et al. [1985]:

$$f(r) = \begin{cases} b(1 - 3r^2 / d^2), & 0 < r \leq d / 3 \\ 1.5b(1 - r / d)^2, & d / 3 < r \leq d \\ 0, & r > d \end{cases}$$

$r$  is distance to the skeleton point,  
 $d$  is its radius of influence



# Soft objects

---

G. Wyvill et al. [1986]:

$$f(r) = \begin{cases} 1 - \frac{22r^2}{9d^2} + \frac{17r^4}{9d^4} - \frac{4r^6}{9d^6}, & 0 < r \leq d \\ 0, & r > d \end{cases}$$



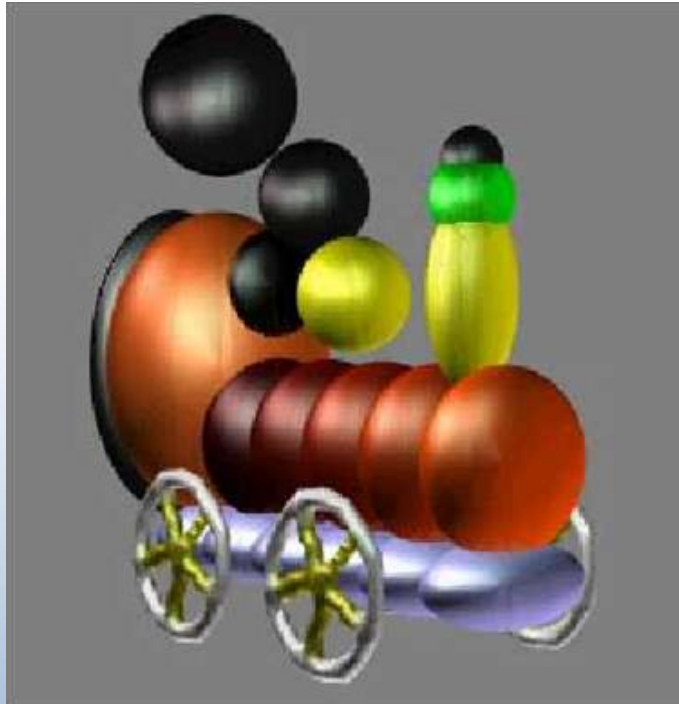
# Other Skeletal Elements

---

- Straight line segment
- Parametric curve
- Polygon
- Parametric surface
  - Numerical distance calculation
  - Unwanted bulges



# The Soft Train (1986)



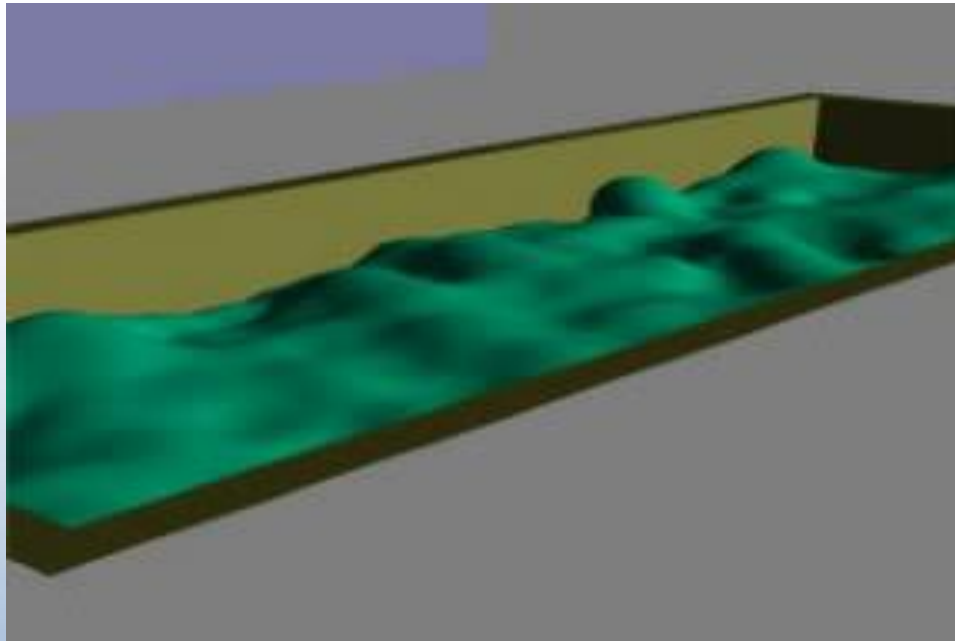
$$\mathbf{F}_{\text{total}}(\mathbf{P}) = \sum_{i=1}^{i=n} \mathbf{c}_i \mathbf{F}_i(\mathbf{r}_i)$$

Slide courtesy of Brian Wyvill



# SOFT (SIGGRAPH 1986)

---



Courtesy of Brian Wyvill





# Convolution Integral

---

$$f(X) = \int_{R^3} s(P)h(X - P)dP$$

- $s(X)$  is a predicate function defining geometry of the skeletal element
  - $h(X)$  is a convolution kernel
- Heavy numerical calculations



# Superposition of Convolutions

---

$$h \otimes (s_1 + s_2) = h \otimes s_1 + h \otimes s_2$$

$\otimes$  means convolution

•  $s_1 + s_2$  means union of skeletal elements

$h$  is the kernel



# Analytical Solutions

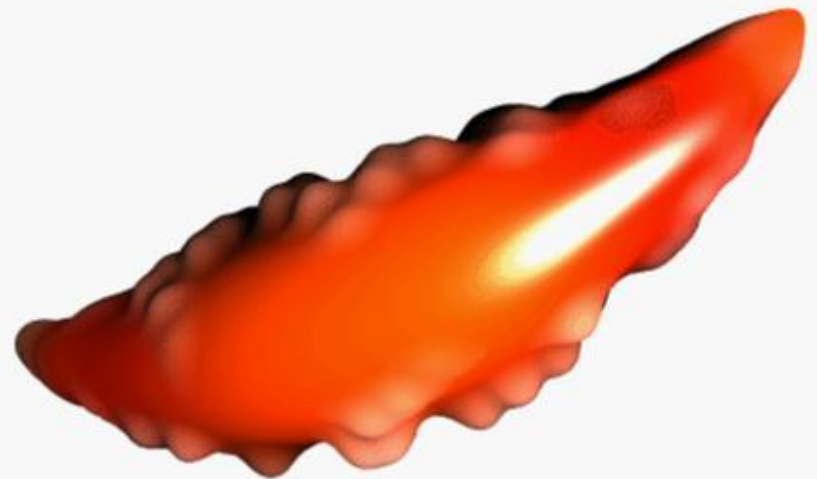
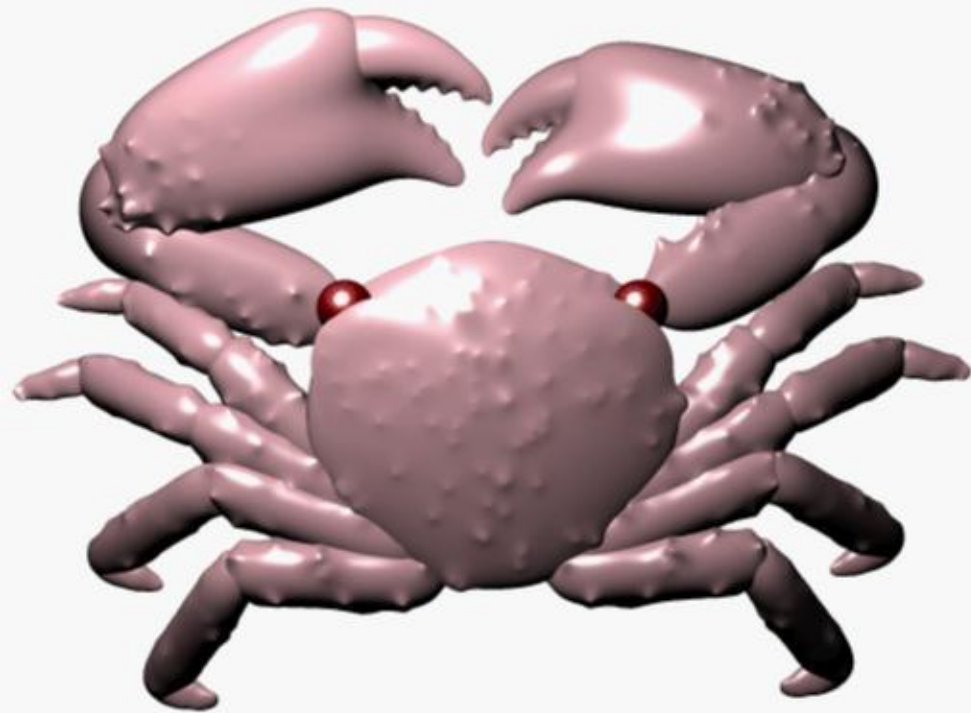
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$$h(X) = 1 / (1 + a^2 \|X\|^2)^2$$

Sherstyuk [1998]:

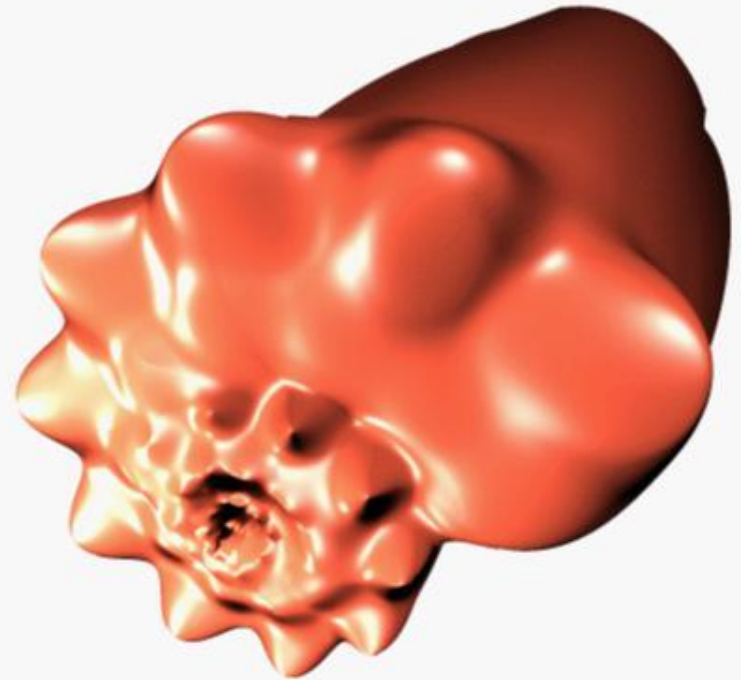
- Point
- Segment
- Circular arc
- Triangle
- Plane

# Convolution surfaces by A. Sherstyuk [SMI99]



Images courtesy of Andrey Sherstyuk

# Convolution surfaces by A. Sherstyuk [SMI99]



Images courtesy of Andrey Sherstyuk



# Advantages of Skeletal Models

---

- Intuitive representations for *natural* objects (molecular shapes, liquid and melting objects, animal and human body);
- Automatic shape *blending*;
- Skeletons are easily manipulated and displayed;



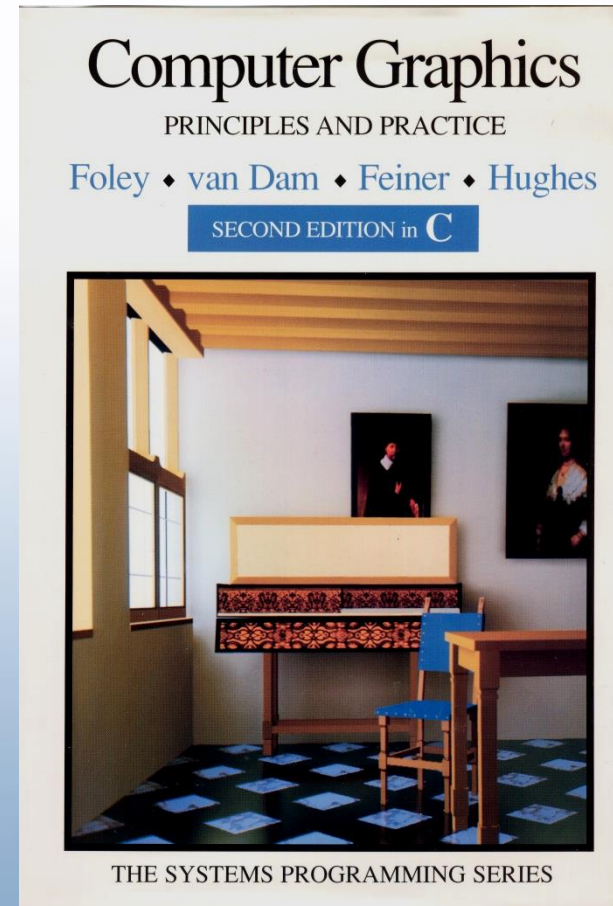
## Advantages of Skeletal Models

- More concise representation than parametric surfaces;
- Complex shapes can be modeled with few primitive elements;
- Easy to add new primitive types by defining a new skeleton or a new field function.



# References

- James D. Foley, Andries van Dam, Steven K. Feiner, John F. Hughes, Computer Graphics: Principles and Practice (2nd Edition in C ), Addison-Wesley, Reading, MA, 1997.







# References

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- Bloomenthal J., Shoemake K., **Convolution surfaces**, SIGGRAPH'91, Computer Graphics, vol.25, No.4, 1991, pp.251-256.
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